

L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2025-2026, week 4
PL exercises

Exercises related to “Peer-to-peer learning/teaching” (P2P L/T)

P2P Exercise 1

A measure of the center of gravity of the sequence $x[n]$ may be estimated using

$$C_G = \frac{\sum_{n=-\infty}^{+\infty} n x[n]}{\sum_{n=-\infty}^{+\infty} x[n]}. \quad (1)$$

Using the properties of the Fourier Transform, show that this center of gravity measure may also be estimated in the frequency domain using

$$C_G = \frac{j \left. \frac{dX(e^{j\omega})}{d\omega} \right|_{\omega=0}}{X(e^{j\omega})|_{\omega=0}}$$

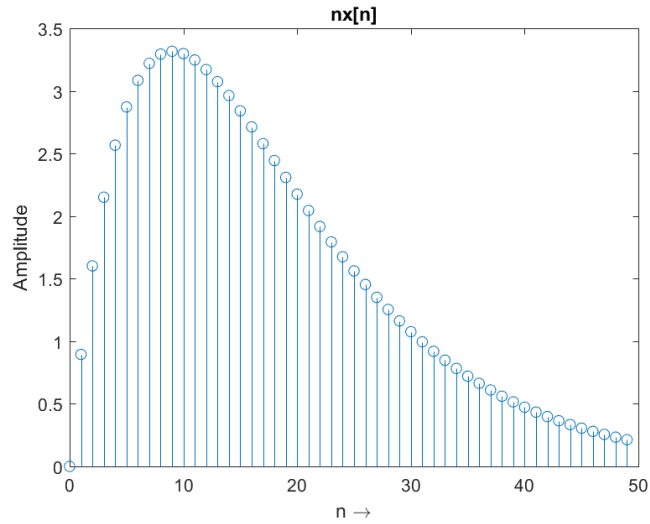
Using this result and considering that $x[n] = a^n u[n]$, show that $C_G = \frac{a}{1-a}$.

P2P assessment: 3pt /5 if demonstration is complete and without errors

Using this compact equation, find C_G in case $a = 0.895$. We take this as our reference result.

Now, using Equation (1), estimate C_G in Matlab/Octave when the number of samples is 50, or 100. Are the numerical results consistent with our reference result ?

Just as a reference, a plot of $nx[n]$ is as follows.

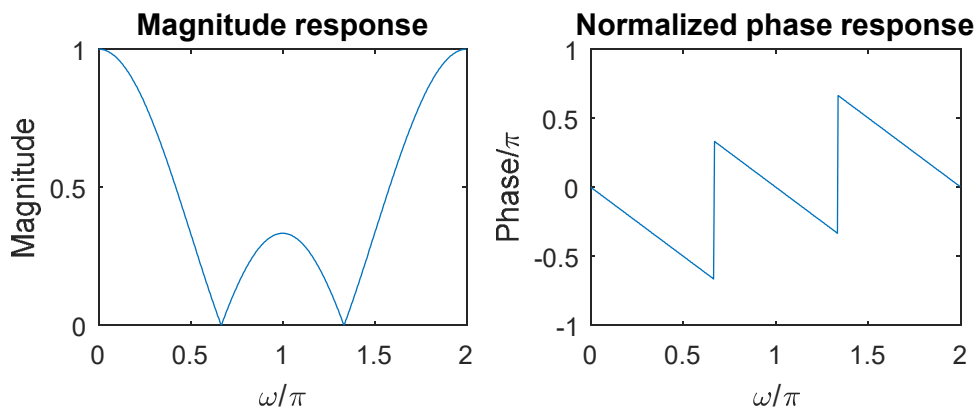


P2P assessment: 2pt /5 if Matlab/Octave code is implemented in such a way as to generate the above figure and if reference $C_G \cong 8.524$ and if the numerical results are shown to be approximately 8.328 and 8.5223.

P2P Exercise 2

A discrete-time system consists of a moving average filter of length 3, i.e., its impulse response can be expressed as $h[n] = (u[n] - u[n - 3])/3$.

Show in Matlab/Octave that (using `freqz()` and `angle()` commands, or equivalent) the magnitude and phase representation of the frequency response of this filter, $H(e^{j\omega})$, is as illustrated next.



P2P assessment: 2pt /5 if plots as in the above figures are generated in Matlab/Octave

Show that the compact analytical expressions that reflect those magnitude and phase responses are, respectively:

$$|H(e^{j\omega})| = \frac{1}{3} \frac{\left| \sin \frac{3\omega}{2} \right|}{\left| \sin \frac{\omega}{2} \right|} = \frac{\left| \text{sinc} \frac{3\omega}{2} \right|}{\left| \text{sinc} \frac{\omega}{2} \right|}$$

and

$$\angle H(e^{j\omega}) = -\omega \pm \pi \text{ jumps (when } \frac{\sin \frac{3\omega}{2}}{\sin \frac{\omega}{2}} \text{ switches polarity),}$$

and show how these expressions give rise to the above graphical representations.

P2P assessment: 3pt /5 if demonstration is complete and without errors

Note: in this exercise, we are using the convention $\text{sinc } \alpha = \frac{\sin \alpha}{\alpha}$, in Matlab the convention is $\text{sinc } \alpha = \frac{\sin \pi \alpha}{\pi \alpha}$

Exercise extension:

If the sampling frequency is 1 kHz, what is the frequency, in Hertz, of a real-valued sinusoid that is exactly blocked by this filter ?

You can demonstrate graphically this using the following Matlab code:

```
FS=1000; TS=1/FS; Freq= ???
t=[0:TS:25E-3];
x=sin(2*pi*Freq*t);
h=ones(1,3)/3;
y=conv(h, x);
figure(1); subplot(2,1,1)
plot(t,x); ylabel('Amplitude'); title('Input')
subplot(2,1,2)
plot(t,y(1:length(x)))
xlabel('Time (s)'); title('Output')
```

Discuss possible transient effects when the signal starts.