

L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2025-2026, week 5
TP (Recitation) exercises

Topics: the Z-Transform

Exercise 1

Using the definition, obtain the Z transform of the following signals:

- a) $\delta[n]$
- b) $u[n]$
- c) $a^n u[n]$
- d) $\cos(n\omega)u[n]$

Exercise 2

If $X(z)$ is the Z transform of the discrete-time signal $x[n]$, obtain the signals whose transforms are:

- a) $X(-z)$
- b) $X(z^2)$
- c) $X(z^{-1})$
- d) (OPTIONAL) Use a microphone correctly plugged to your computer and the following Matlab code (or an alternative audio recorder) to record the sound of your voice during 10 seconds. During this recording time you should utter different vowels, for example, a-e-i-o-u, as well as your complete name. This Matlab code sets the sampling frequency to 22050 Hertz and sets the sample resolution to 16 bits.

```
FS=22050; duration=4; NBITS=16;
r = audiorecorder(FS, NBITS, 1);
fprintf('Start speaking...\n');
record(r); % speak into microphone...
pause(duration); stop(r);
% p = play(r); % listen to complete recording
x = getaudiodata(r, 'single'); % get data normalized to +/-1.0
x=x(FS:end); % avoid first second, may contain noise
fprintf('Stop speaking. Now playing...\n');
sound(x,FS);
fprintf('Stop playing.\n');
audiowrite('ficheiro.wav', x, FS, 'BitsPerSample', NBITS);
```

Now, add also the following Matlab code which is used to play the recorded sound file:

```
[x,FS,NBITS]=wavread('soundfile.wav'); % or
[x,FS]=audioread('soundfile.wav');
sound(x,FS); %NOTE: x values are in the range [-1, 1]
N=length(x);
samples=[0:N-1];
figure(1)
plot(samples/FS, x);
xlabel('Time (s)');
ylabel('Amplitude');
title('soundfile.wav');
```

In Matlab, create new vectors x_a , x_b and x_c , corresponding respectively to **a)**, **b)** and **c)**, as above. The latter should include an additional modification: $z^{-(N-1)}X(z^{-1})$. As suggested in the Matlab code, play and listen to the audio resulting from x_a , x_b and x_c (after making sure that the range in each case is constrained to $[-1, 1]$) and try to understand the relation between the sound of each signal and the signal transformation that is implemented in each case.

Suggestion: in Matlab try `help upsample`

Exercise 3

Find $y[n]$, the output of the discrete-time and causal system characterized by the transfer function

$$H(z) = \frac{2}{1 + 0.4Z^{-1}} \text{ when its input is given by } x[n] = u[n],$$

- a) by computing the discrete convolution between $x[n]$ and the impulse response $h[n]$,
- b) by computing the inverse Z transform of $Y(z)$,

compare the “analytical” result obtained in **a)** or **b)** (if analytical results look different manipulate one of them such that it equals the other) with the numerical result in Matlab for $n=0, 1, \dots, 49$.

Hint: in Matlab try `help impz`