

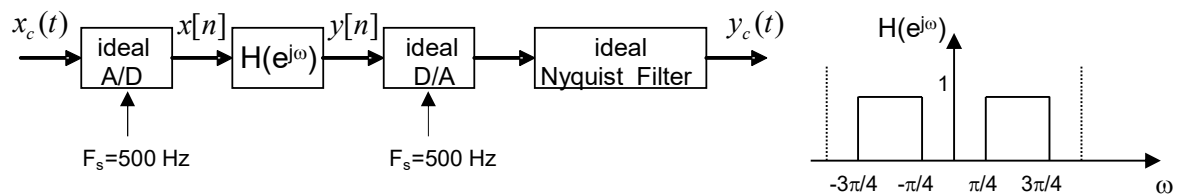
L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2025-2026, week 6
P2P exercises

Exercises related to “Peer-to-peer learning/teaching” (P2P L/T)

P2P Exercise 1

The continuous-time signal $x_c(t) = 1 + \cos(200\pi t) + \sin(700\pi t)$ excites the following system where the sampling frequency is 500 Hz. $H(e^{j\omega})$ represents an ideal band-pass filter whose pass-band is defined in the range $\pi/4 \leq |\omega| \leq 3\pi/4$, as illustrated. Notice that an *anti-aliasing* filter does *not* exist.



- a) What are the frequencies, in Hertz, of the analog input signal ?

Note: the answer should be 100 Hz and 350 Hz (do they exceed the Nyquist range ?).

- b) If a suitable *anti-aliasing* filter existed, what would the equivalent analog system be between $x_c(t)$ and $y_c(t)$ (i.e. from an end-to-end point of view) ?

Note: the answer should be: an analog band-pass filter having cutoff frequencies 62.5 Hz and 187.5 Hz (why ?).

P2P assessment: 1pt /5 if explanation is clear and complete

- c) Find the frequencies (in the Nyquist range, i.e. with $-\pi \leq \omega < \pi$) that the discrete-time signal $x[n]$ contains, and write a compact expression for $x[n]$.

Note: the answers should be $\omega_0 = 0$ rad., $\omega_1 = 2\pi/5$ rad., $\omega_2 = -3\pi/5$ rad., and $x[n] = 1 + \cos\left(n\frac{2\pi}{5}\right) - \sin\left(n\frac{3\pi}{5}\right)$.

P2P assessment: 2pt /5 if analytical results are correct and explanation is clear and complete

- d) Considering ideal reconstruction, find an expression for $y_c(t)$.

Note: the answer should be $y_c(t) = \cos(200\pi t) - \sin(300\pi t)$.

P2P assessment: 2pt /5 if analytical result is correct and explanation of output frequencies is clear

P2P Exercise 2

Consider the ideal frequency response of the discrete-time filter as specified in P2P exercise 1.

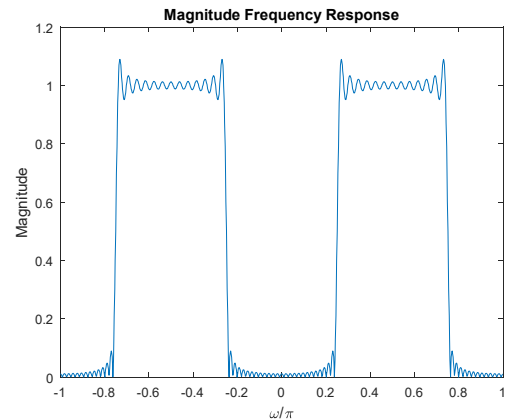
- a) Find the ideal impulse response of the filter, $h[n]$.

Note: the answer should be $h[n] = \frac{2}{n\pi} \left(\cos\left(\frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{4}\right) \right) = \frac{1}{n\pi} \left(\sin\left(\frac{n3\pi}{4}\right) - \sin\left(\frac{n\pi}{4}\right) \right)$

P2P assessment: 3pt /5 if analytical result is correct and explanation is clear and complete

- b) Complete the following Matlab code in order to obtain the frequency response that is displayed next

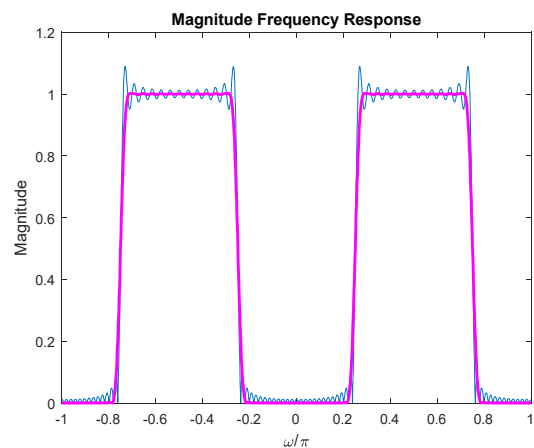
```
n=[-50:50];
wk=[-pi:pi/512:pi-pi/512];
h= TO BE COMPLETED ;
h(51)= TO BE COMPLETED ;
[H W]=freqz(h,1,wk);
plot(W/pi, abs(H))
xlabel('\omega/\pi')
ylabel('Magnitude')
title('Magnitude Frequency Response')
```



P2P assessment: 1pt /5 if Matlab/Octave code correct and obtained figure is as shown

- c) Try to explain what the following code does from the point of view of modification of the impulse response in the discrete-time domain, and interpret the corresponding implication in the frequency domain.

```
win=hamming(length(h)).';
hmod=h.*win;
[Hmod W]=freqz(hmod,1,wk);
hold on
plot(W/pi, abs(Hmod), 'm')
hold off
```



- d) Show how this band-pass filter can be obtained as a transformation of a low-pass filter.

Hint: Consider how the frequency response of a low-pass filter could be used to produce the above frequency response of the band-pass filter.

Note: the answer should be: first, a low-pass filter having $\omega_c = \pi/4$ rad. is designed, then it is modulated to $\omega_M = \pm\pi/2$ rad (the analytical demonstration of this is required).

P2P assessment: 1pt /5 if explanation is clear and if connection to the analytical result of 2a) is made