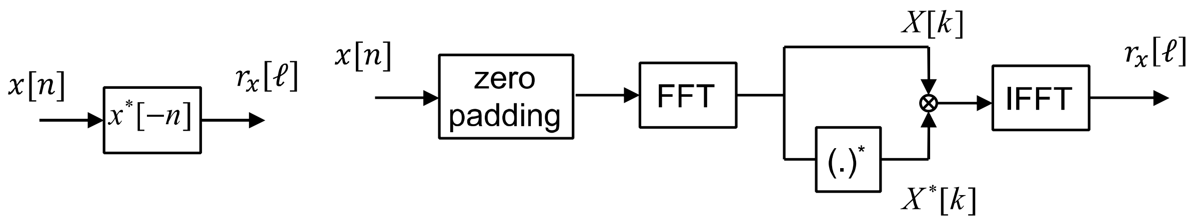
L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2025-2026, weeks 12-13
TP (Recitation) exercises

Topics: Computation of the auto-correlation function using the DFT. Time-domain windowing in the frequency-domain. The magnitude spectrum (periodogram) and the spectrogram.

Exercise 1

According to the Wiener-Khintchine theorem, the following two block diagrams represent two possible alternatives to compute the auto-correlation of a signal $x[n]$ that we admit is real-valued and has length L. Consider that both FFT and IFFT have length N, where N is a power of two number.

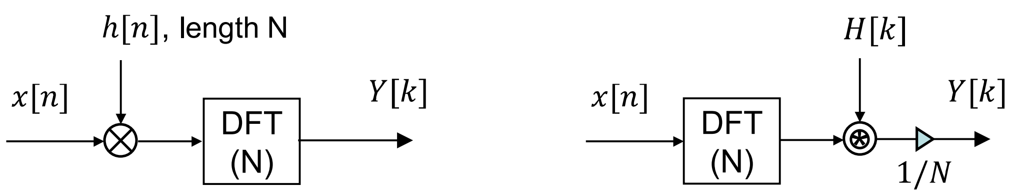


a) Clarify under which condition both alternatives yield the same desired result.

b) Admitting that N=2L, find the lowest value of L above which the second alternative (using FFT) is more advantageous than the first one from the point of view of computational complexity. Consider just multiplication operations and consider that that the computational cost of a complex multiplication is equivalent to four real multiplications.

Exercise 2

As discussed in recent classes, windowing is an important signal processing step helping to control the leakage phenomenon in DFT-based spectrum analysis. Most often, windowing takes place in the discrete-time domain, as represented on the left-hand side of the following illustration. However, in certain cases, the equivalent operation must be performed in the frequency domain as illustrated on the right-hand side. In this case, it is desired that $H[k]$, the DFT of $h[n]$, has a compact support, i.e. that only a few coefficients are non-zero (why ?).



a) As can be concluded from the developments in the guide concerning this week's Lab, the standard Hamming or Hanning window do not have a compact support. However, the *shifted* Hanning window has a compact support. It is defined as

$$h_{Hann}[n] = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi}{N}\left(n + \frac{1}{2}\right)\right)\right], \quad n = 0,1,\dots,N-1.$$

Find $H[k]$, the DFT of $h_{Hann}[n]$.

b) Using $H[k]$, the signal and N parameter of P2P Exercise 1 of week 13, complete the following Matlab code to check the equivalence between windowing in the time domain, and the corresponding operation in the frequency domain.

Hint 1: recall the DFT properties

Hint 2: the circular convolution is implemented in Matlab using `cconv()`

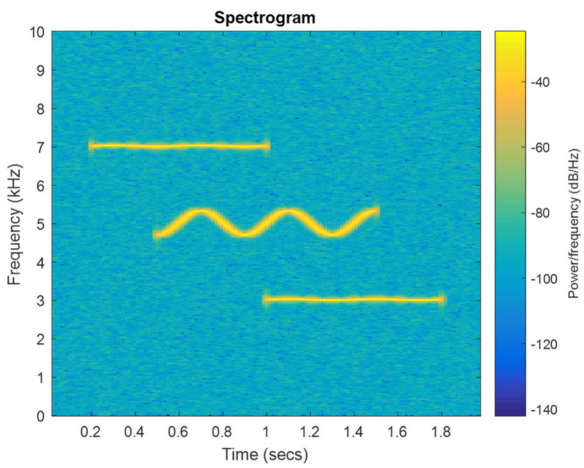
```
N=16; n= [0:N-1];
h=0.5*(1-cos(2*pi/N*(n+0.5)));
x=1+sin(n*2*pi/N).*cos(n*4*pi/N);
y=x.*h;
Y=fft(y);

H = zeros(1,N);
H(1)= ... ;
H( )= ... ;
...
X=fft(x);
Z=... ;
stem(abs(Y-Z)) % this error should be small < 1E-8
```

Exercise 3

Download from the Moodle platform the WAV file `audioSPEC.wav`. Then, execute the following Matlab commands to obtain the spectrogram of the signal contained in that file.

```
inpfile='audioSPEC.wav';
[x, FS]=audioread(inpfile);
N=1024; shift=N/4;
spectrogram(x,hann(N),N-shift,N,FS,'yaxis')
title('Spectrogram')
```



Based on this spectrogram, interpret the signal activity it reveals.