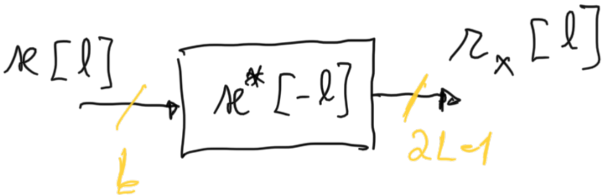
FPS 12jan2022

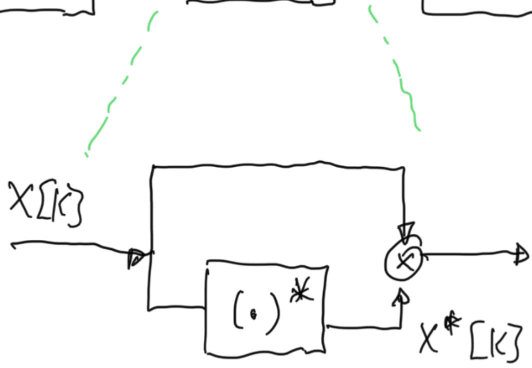
TP class

$\boxed{1}$



$$r_x[l] = x[l] * x^*[-l] = x[l] * x[-l]$$

$$= \sum_{k=-\infty}^{+\infty} x[k]\, x[-(l-k)] = \sum_{k=-\infty}^{+\infty} x[k]\, x[k-l]$$



a) $N \geq 2L-1$

b) $N = 2L$

(multiplications only)

$$1 + 2 + \dots + (L-1) + L + (L-1) + \dots + 2 + 1$$

$$= 2\sum_{k=1}^{L-1} k + L = \cancel{2}\,\frac{(L-1)L}{\cancel{2}} + L = L^2 - L + L = L^2$$

(we could also reach this conclusion consider
polynomial product in the Z-domain

$$(a+jb)(c+jd) = ac - bd + j(bc+ad)$$

in the frequency-domain:

$$4 \times 2 \times \frac{N}{2} \log_2 N + 4 \times N$$

advantage if:

$$4\cancel{N}\log_2 N + 4\cancel{N} < L^2 = \left(\frac{N}{2}\right)^2 = \frac{N^{\cancel{2}}}{4}$$

$$\Leftrightarrow \quad 16\,(1 + \log_2 N) < N$$

N	$16(1+\log_2 N)$
32	96
64	112
128	128
256	144

✓ OK!

if $L \geq 128$ it is preferable to compu
$r_x[l]$ using frequency-domain
processing, i.e. using FFT!