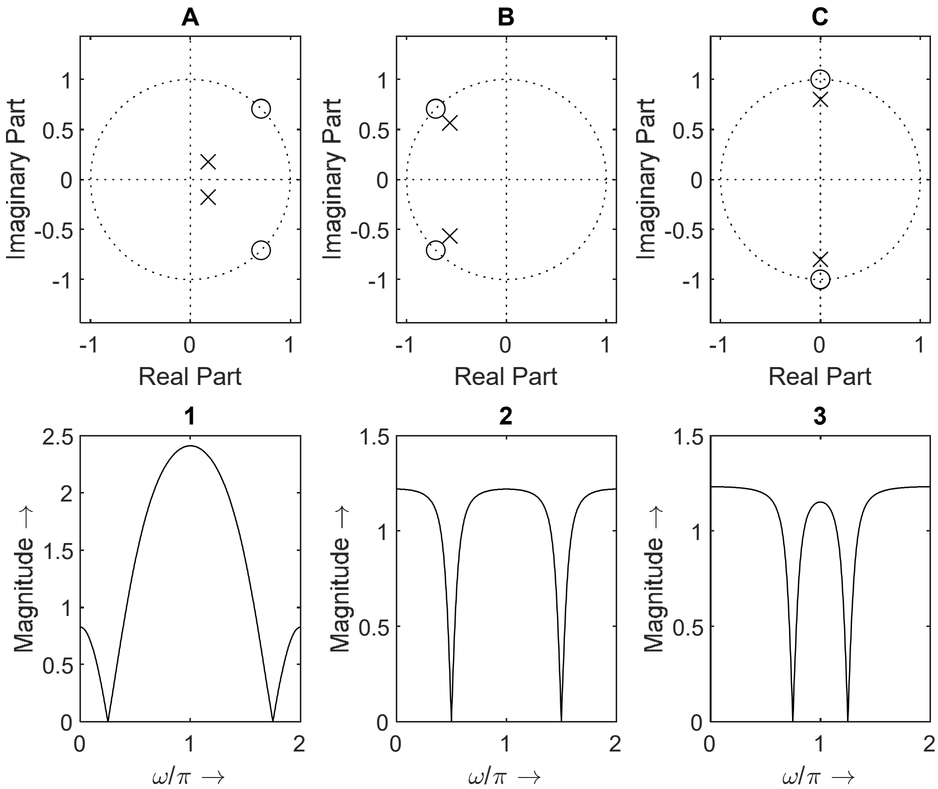
L.EEC/M.EEC
L.EEC025 - Fundamentals of Signal Processing

FIRST EXAM, JANUARY 14, 2026
Duration: 120 Minutes, closed book

NOTE: each question ***must*** be answered in a separate sheet; please write your name and Student order number on all sheets, please provide complete answers while trying to minimize paper usage. Except for a basic scientific calculator and the provided formulae sheet, no other materials or tools, including (so-called) AI assistants, are allowed in this exam.
DISCLAIMER: no FunSP materials, including this exam, have been produced using any (so-called) AI-based tools.

1. Three different causal discrete-time systems have zero-pole diagrams A, B, and C, and the frequency response magnitudes 1, 2, and 3, as depicted next. The radius of each represented pole, or zero, is either 0.25, 0.8 or 1.0, and the angles of the off-axis poles, or zeros, are $\pm\pi/4$ rad or $\pm 3\pi/4$ rad.



a) **[1,5 *pts*]** Match each zero-pole diagram (A, B, C) to the corresponding frequency response magnitude (1, 2, 3), and indicate the main supporting arguments.

b) **[1 *pt*]** Characterize systems A, B, and C from the point of view of order, stability, and real-valued/complex-valued impulse response.

c) **[1 *pt*]** Admit that system A is cascaded with system B. Sketch a possible frequency response magnitude. Explain briefly.

d) **[1 *pt*]** Consider the following statement: « By scaling the zeros and poles of system A by the same factor, a non-trivial all-pass system is obtained». Is this true or false ? If true, explain how the impulse response of system A, $h_A[n]$, should be modified to reflect that scaling.

2. One of the STM32F7-based LAB assignments, in this FunSP edition, consisted of signal generation using a lookup table. A table of a sampled sinewave was constructed using sine_table $= G \times \sin(n\omega_0 + \theta)$, were G is a constant, ω_0 is the frequency, and θ is the starting phase. The sampling frequency was set to 8000 Hz, and the table was programmed as:

```
#define LOOPLENGTH 6
int16_t sine_table[LOOPLENGTH] = {0, 8660, 8660, 0, -8660, -8660};
```

a) **[1 *pt*]** Find the value of ω_0 (in rad.) as well as the frequency (in Hertz) of the generated analog wave.

b) **[0.5 *pts*]** If `tx_sample_L` and `tx_sample_R` represent the sample values on the Left and Right output channels, respectively, and the main code in the Interrupt Service Routine, consists of:

```
tx_sample_L = sine_table[sine_ptr_L];
tx_sample_R = sine_table[sine_ptr_R];
sine_ptr_L = (sine_ptr_L+1)%LOOPLENGTH;
sine_ptr_R = (sine_ptr_R+1)%LOOPLENGTH;
```

indicate how the `sine_ptr_L` and `sine_ptr_R` pointers should be initialized so that the phase shift between the two output analog waves is $\pi/3$ rad.

c) **[1 *pt*]** If, in the code in **b)** the pointer increment is changed from `sine_ptr_L+1` to `sine_ptr_L+2`, what is the consequence to the generated analog wave ?

d) **[1 *pt*]** If the code in **b)** is preserved but the table initialization is changed to:

```
#define LOOPLENGTH 4
int16_t sine_table[LOOPLENGTH] = {8660, 8660,-8660, -8660};
```

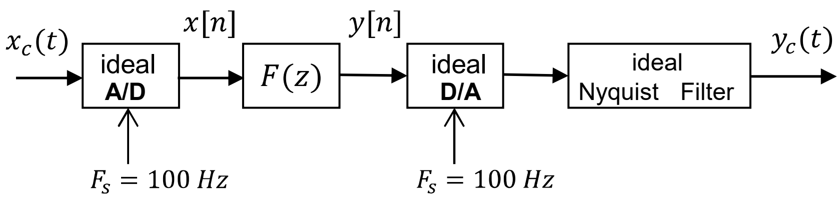
What are the main consequences to the generated analog wave (in either Left or Right channel) ?

3. Consider the discrete-time system C whose zero-pole diagram is represented in Prob. 1. Admit that it is causal. The radius of the poles is $r = 0.8$.

a) **[1,5 *pts*]** Find the transfer function of the system, $H(z)$, write a difference equation implementing it, and sketch a corresponding canonic realization structure.

b) **[1 *pt*]** Obtain a compact expression describing the magnitude of the frequency response of the system, $\left|H\left(e^{j\omega}\right)\right|$, and show that its gain for $\omega = 0$ rad., or $\omega = \pi$ rad., depends on $\frac{2}{1+r^2}$.

c) **[1 *pt*]** Consider the illustrated mixed analog and discrete-time system where $F(z) = \frac{1+z^{-1}}{1+rz^{-1}}$, $|z| > r$. An *anti-aliasing* filter does not exist and the analog input signal is $x_c(t) = 1 + \sin(350\pi t)$.



Find the sinusoidal frequencies of the discrete-time signal $x[n]$ in the Nyquist range, i.e., in the range $-\pi \le \omega < \pi$. Obtain a compact expression for $x[n]$.

d) **[1,5 *pts*]** Using $F(e^{j\omega})$ obtain $y[n]$ and, admitting ideal reconstruction, obtain $y_c(t)$.

4. Consider the following Matlab code.

```
x=[1  2  3  4+1j  1j  1j];
X=fft(x); N=length(X);
Y=zeros(size(X)); Y(1)=conj(X(1));
Y(2:N)=conj(X(N:-1:2));
A=(X+Y)/2; B=(X-Y)/2; C=A.*B;
ifft(C)
```

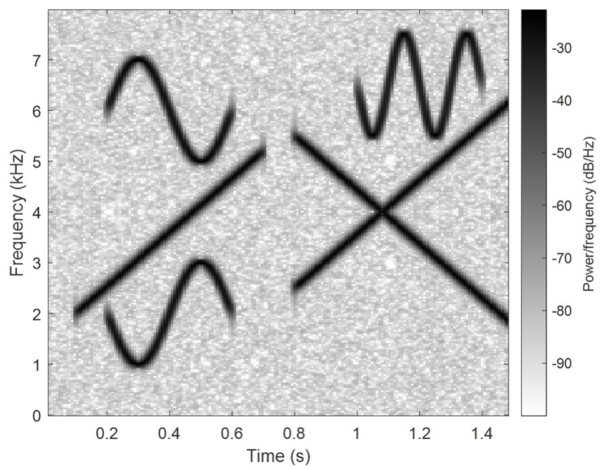
a) **[1 *pt*]** Find the result of `ifft(Y)` without executing the code.

b) **[1 *pt*]** Without executing the code, express the result of `ifft(A)` and of `ifft(B)` as a function of `x[n]`.

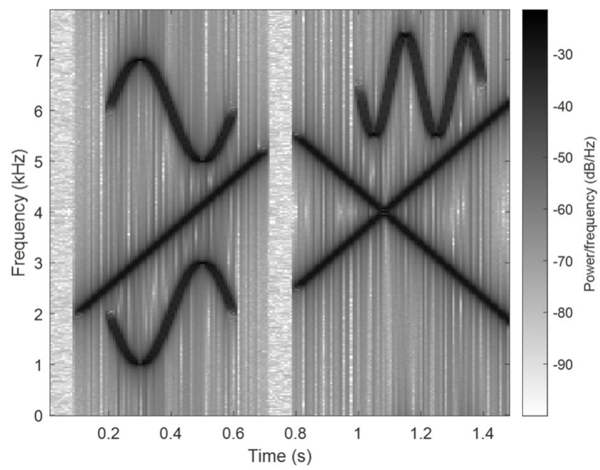
c) **[1,5 *pts*]** Find the result of `ifft(C)` without executing the code.

5. The spectral contents of a synthetic signal sampled at F_S=8000 samples/s was analyzed by means of spectrograms. The signal contains two real-valued and two complex-valued signal components. The starting frequencies of the real-valued signal components are within the Nyquist range, and all frequencies evolve *monotonously* over time. The represented spectrograms (A, B) were obtained using two alternative windows (Rectangular and Hanning), and a sliding FFT (N=256) with 50% overlap between adjacent FFT.

A



B



a) **[1 *pt*]** What is the Nyquist frequency ? What window has been used to generate spectrogram A and spectrogram B ? Explain your reasoning.

Note: the blurred effects in the spectrograms reflect the impact of signal processing, not printer problems.

b) **[1 *pt*]** Represent two plausible periodograms (or power spectra), one corresponding to `t=0.5 s`, and another to `t=1.2 s`. Explain.

c) **[1,5 *pts*]** Based on the observation of the spectrograms, describe the spectral contents of the synthetic signal.

END