U.PORTO
FEUP FACULDADE DE ENGENHARIA
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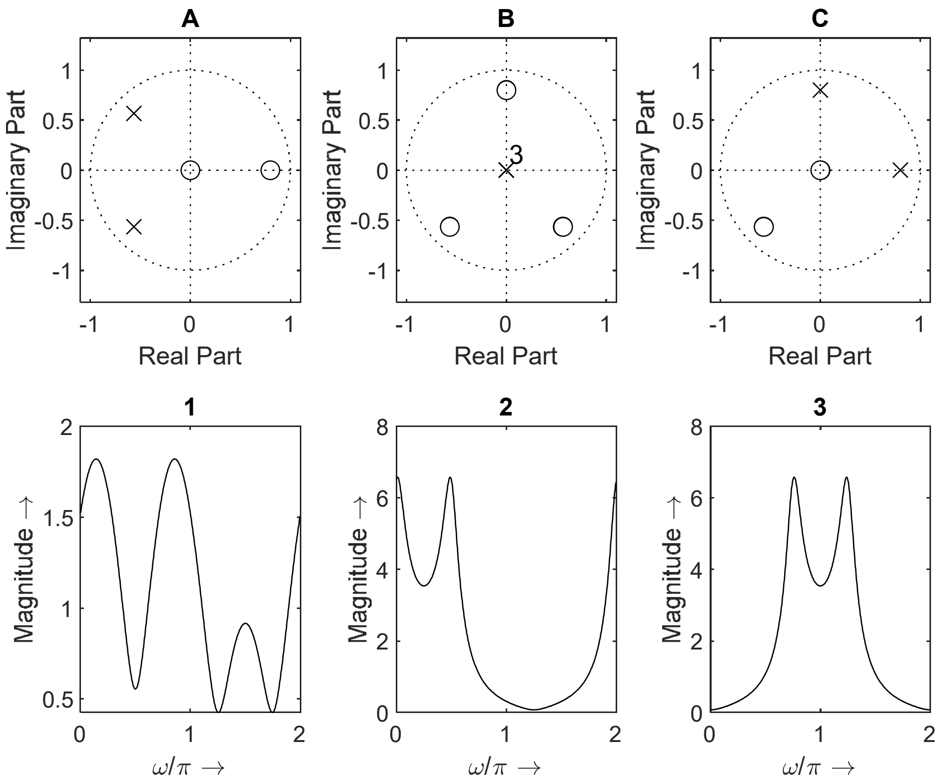
# L.EEC/M.EEC
# L.EEC025 - Fundamentals of Signal Processing

**SECOND EXAM, JANUARY 30, 2026**
**Duration: 120 Minutes, closed book**

**NOTE**: each question ***must*** be answered in a separate sheet; please write your name and Student order number on all sheets, please provide complete answers while trying to minimize paper usage. Except for a basic scientific calculator and the provided formulae sheet, no other materials or tools, including (so-called) AI assistants, are allowed in this exam.
**DISCLAIMER**: no FunSP materials, including this exam, have been produced using any (so-called) AI-based tools.

**1.** Three different causal discrete-time systems have zero-pole diagrams A, B, and C, and the frequency response magnitudes 1, 2, and 3, as depicted next. The radius of each represented pole, or zero, is 0.0 or 0.8, and the angles of all poles, or zeros, are multiples of $\pi/4$ rad.




**a)** **[1,5 *pts*]** Match each zero-pole diagram (A, B, C) to the corresponding frequency response magnitude (1, 2, 3), and indicate the main supporting arguments.

**b)** **[0,5 *pts*]** Characterize systems A, B, and C from the point of view of order, stability, and real-valued/complex-valued impulse response.

**c)** **[1 *pt*]** Sketch the zero-pole diagram of the inverse system of system B, and sketch a possible frequency response magnitude. Explain briefly.

**d)** **[1,5 *pts*]** Consider the following statement: «By combining systems A and B in a suitable way, an FIR system results». What combination is that, and what is the resulting system order ? If $h_A[n]$ and $h_B[n]$ represent the impulse responses of systems A and B, respectively, express that combination as a function of $h_A[n]$ and $h_B[n]$.

**2.** One of the STM32F7-based LAB assignments, in this FunSP edition, involved the implementation of a moving average filter for real time operation. The code initialization included setting `N=5`, `invN=1/N`, `yn_1=0.0`, and memory allocation to vectors `h[N]` and `x[N+1] = {0, 0 ,0, 0, 0, 0}`. The sampling frequency was set to 8000 Hertz.

a) **[0.5 *pts*]** The initial part of the `main()` function included the following (incomplete) code line:

```
for (i=0 ; i<N ; i++) h[i] =
```

Complete the `code: h[i] =` ? Explain briefly.

b) **[1,5 *pts*]** Taking `rx_sample_L` as the sample value of the Left input channel, and `tx_sample_L` and `tx_sample_R` as the sample values of the Left and Right output channels, respectively, the relevant code of the interrupt service routine was:

```
float32_t yn = 0.0;
x[0] = rx_sample_L;
for (i=0 ; i<N ; i++) yn += h[i]*x[i];
tx_sample_L = yn;
yn = (x[0]-x[N]) * invN + yn_1;
tx_sample_R = yn;
for (i=N ; i>0 ; i--) x[i] = x[i-1];
yn_1 = yn;
```

Write down the difference equations used to generate the output of the Left and Right channels and clarify if the Left and Right analog outputs are different or not.
**[Hint**: use z-Transform analysis]

c) **[1,5 *pts*]** Obtain the frequency response magnitude of the discrete-time system associated with the Right channel (whose difference equation is implemented by the code line `yn = (x[0]-x[N]) * invN + yn_1` ) and indicate what analog frequencies (in Hertz) are blocked by the system.

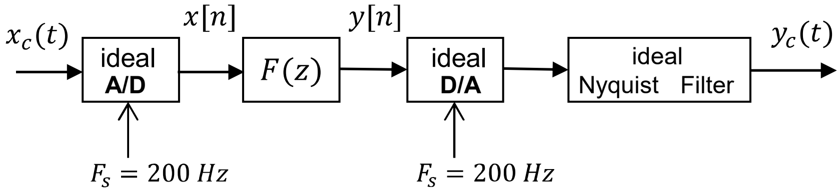
d) **[1 *pt*]** Admit that $H(z)$ represents the transfer function of the discrete-time system implemented in the Right channel (whose difference equation is implemented by the code line `yn = (x[0]-x[N]) * invN + yn_1` ). Explain how the code should be modified such as to implement $H(z/0.8)$.

3. Consider that the transfer function of a discrete-time system is $F(z) = \frac{1+rz^{-1}}{1-rz^{-1}}$, $|z| > r$.

a) **[1 *pt*]** Write a difference equation implementing it, and sketch the transposed form of the direct type-2 realization structure.

b) **[1,5 *pts*]** Obtain a compact expression describing the magnitude of the frequency response of the system, $|F(e^{j\omega})|$, and identify the frequencies for which its gain does not depend on $r$.

c) **[1 *pt*]** Consider the illustrated analog and causal discrete-time system whose transfer function is $F(z)$. An *anti-aliasing* filter does not exist and the input analog signal is $x_c(t) = 1 + \sin(500\pi t)$.




Find the sinusoidal frequencies of the discrete-time signal $x[n]$ in the Nyquist range, i.e., in the range $-\pi \le \omega < \pi$. Obtain a compact expression for $x[n]$.

**d) [1,5 *pts*]** Using $F(e^{j\omega})$ obtain $y[n]$ and, presuming ideal reconstruction, obtain $y_c(t)$.

**4.** Consider the following Matlab code.

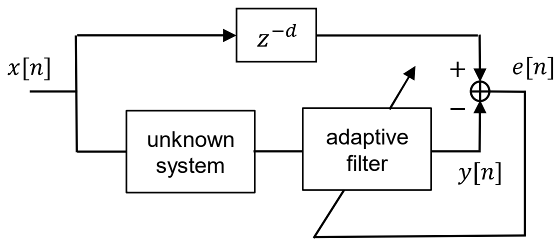
```
x=[1  1j  2  1j  3  1j];
X=fft(x); N=length(x);
Y=X.*conj(X); y=ifft(Y)
A=zeros(size(X)); A(1)=conj(X(1));
A(2:N)=conj(X(N:-1:2));
B=cconv(X,A,N)/N; % cconv()computes the circular convolution
b=ifft(B)
```

**a) [1,5 *pts*]** Find the result of `y=ifft(Y)` without executing the code.

**b) [1 *pt*]** Without executing the code, find the result of `b=ifft(B)`.

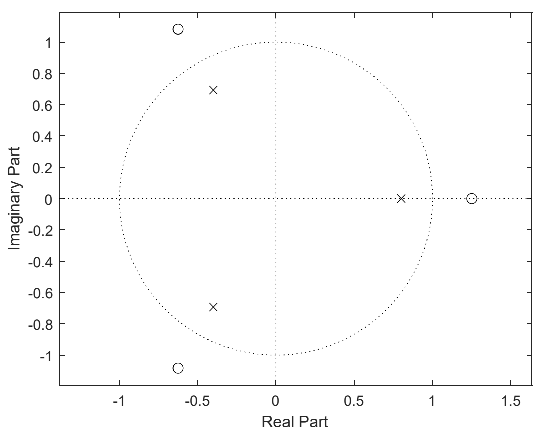
**c) [1 *pt*]** Explain why `y(1) = sum(b)`.

**5.** Consider the illustrated FIR adaptive filter configuration. The LMS adaptation algorithm is used ($h_{n+1}[k] = h_n[k] + \mu\, e[n]x[n-k]$), the adaptation step size is $\mu = 0.005$, and the excitation is white Gaussian noise.

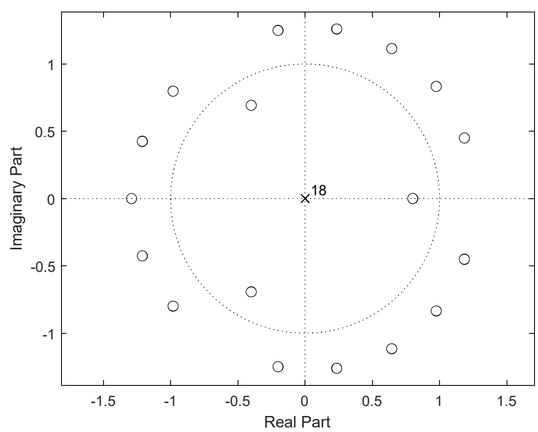



The zero-pole diagram of the unknown system (to the adaptive filter) is represented in Fig. A. Figure B represents the zero-pole diagram of the adaptive system after convergence.

**A**




**B**




**a) [1 *pt*]** Based on the graphical information, including the zero-pole diagrams, justify the type of adaptive filter configuration being used, as well as the order of the FIR filter.

**b) [1 *pt*]** Explain the structure of the constellation of zeros in Figure B and indicate a reasonable criterion to set the $d$ delay parameter (diagram block represented as $z^{-d}$).

**c) [0,5 *pts*]** In general, if the unknown system is a minimum-phase system then, after convergence, the adaptive filter tends to be a maximum-phase system. Explain why.